Midterm 2 Review

Laws of Derivatives :

Sum / Difference : $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

Constant Multiple : $\frac{d}{dx}(kf(x)) = k\frac{d}{dx}(f(x))$

Product : $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$

Quotient : $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Chain : $y = f(u)$, $u = g(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

Core Examples :

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(a^x) = \ln(a)a^x$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{\ln(a)x}$$

$$\frac{d}{dx}(a^{g(x)}) = \ln(a)a^{g(x)} \cdot g'(x)$$

$$\frac{d}{dx}(\log_a|g(x)|) = \frac{g'(x)}{\ln(a)g(x)}$$

Sign Analysis : Finding when $g(x) > 0$ or $g(x) < 0$

1/ Find c such that

A/ $g(c) = 0$ **or** B/ $g(c)$ DNE (discontinuous really)

2/ Draw a number line and mark these points.

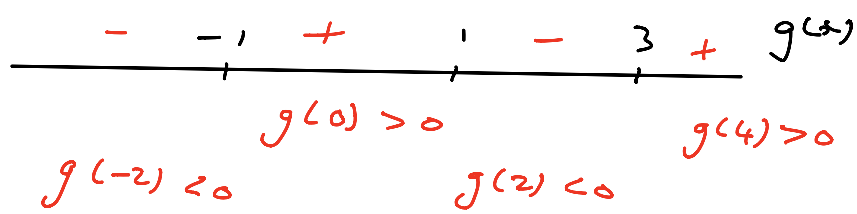
3/ Choose a test point between each and evaluate to see if $+$ or $-$.

Example

$$g(x) = \frac{x^2 - 1}{x - 3}$$

A/ $g(x) = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

B/ g undefined $\Rightarrow x - 3 = 0 \Rightarrow x = 3$



$\Rightarrow$ $g(x) > 0$ on $(-1, 1)$ and $(3, \infty)$

$g(x) < 0$ on $(-\infty, -1)$ and $(1, 3)$

Finding where f increasing / decreasing :

1/ Calculate $f'(x)$

2/ Do sign analysis on $f'(x)$

3/ $f'(x) > 0 \Rightarrow f$ increasing

$f'(x) < 0 \Rightarrow f$ decreasing

Finding relative extrema :

1/ Calculate $f'(x)$

2/ Do sign analysis on $f'(x)$

3/ $f(c)$ exists

$+ \quad c \quad -$ $\quad f'(x) \quad \Rightarrow \quad f(c)$ rel. max

$f(c)$ exists

$- \quad c \quad +$ $\quad f'(x) \quad \Rightarrow \quad f(c)$ rel. min

Finding where f is concave up and concave down :

1/ Calculate $f''(x)$

2/ Do sign analysis on $f''(x)$

3/ $f''(x) > 0 \Rightarrow$ Concave Up

$f''(x) < 0 \Rightarrow$ Concave Down

Finding Inflection Points

1/ Calculate $f''(x)$

2/ Do sign analysis on $f''(x)$.

3/ $f(c)$ exists

$+ \quad c \quad -$ $\quad f''(x) \quad \Rightarrow \quad (c, f(c))$ inflection

$f(c)$ exists

$- \quad c \quad +$ $\quad f''(x) \quad \Rightarrow \quad (c, f(c))$ inflection

Finding absolute extrema on closed interval (f continuous on [a, b])

1/ Calculate $f'(x)$

2/ Find points c such that A/ or B/ hold for $f'(x)$.

3/ Evaluate $f(c)$ for such points and compare

Finding absolute extrema on non-closed interval (f continuous on interval)

1/ Calculate $f'(x)$

2/ Do sign analysis on f'.

3/ If c is only critical number inside interval

$f(c)$ rel. max $\Rightarrow$ $f(c)$ absolute max

$f(c)$ rel. min $\Rightarrow$ $f(c)$ absolute min

Revenue, Cost, Profit

q = number of units

p = price per unit

Demand Equation : $p = D(q)$ ← q independent variable

$$\Rightarrow \underset{\text{Revenue}}{R(q)} = pq = D(q)\,q$$

$$\underset{\text{Profit}}{P(q)} = \underset{\text{Revenue}}{R(q)} - \underset{\text{Cost}}{C(q)}$$

$$\underset{\text{Marginal profit}}{\frac{dP}{dq}} = \underset{\text{Marginal revenue}}{\frac{dR}{dq}} - \underset{\text{Marginal cost}}{\frac{dC}{dq}}$$

$$P'(q) \approx P(q+1) - P(q)$$

$$R'(q) \approx R(q+1) - R(q)$$

$$C'(q) \approx C(q+1) - C(q)$$

Elasticity of Demand

$$\text{Elasticity} = \frac{-p}{q}\,\frac{dq}{dp}$$

$q = D(p)$ ← p independent variable $\Rightarrow$ $E(p) = \frac{-p}{D(p)} D'(p)$

$E(p) > 1 \Rightarrow$ elastic at p $\Rightarrow$ should decrease price to raise revenue

$E(p) < 1 \Rightarrow$ inelastic at p $\Rightarrow$ should increase price to raise revenue

$E(p) = 1 \Rightarrow$ unit elasticity at p $\Rightarrow$ potential location for max revenue.

Constrained Optimization :

1/ Identify objective.

2/ Draw picture, label unknowns

3/ Write objective in terms of unknowns. Gives Objective Formula

4/ Identify constraint. Express it as Constraint Equation in terms of unknowns.

5/ Solve constraint equation in on unknown and sub into objective. Gives f, a single variable function.

6/ Find appropriate domain by looking at f, the picture and the constraint. Find absolute max / min.